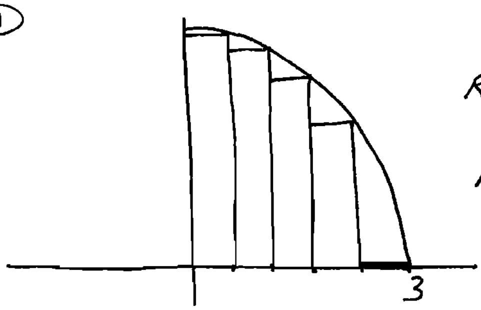
Area approx and exact
Answer Key

1



$\Delta x = \frac{3}{5}$

Right end pts: $\frac{3}{5}i$

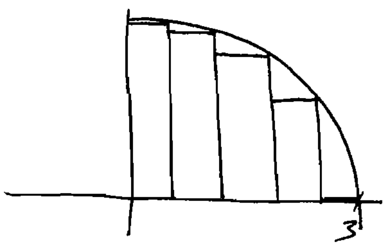
height: $f(\frac{3}{5}i) = -\frac{9i^2}{25} + 9 = \frac{-9i^2+225}{25}$

$$\frac{3}{5}\sum_{i=1}^{5} \frac{-9i^2+225}{25} = \boxed{15.12\ u^2}$$

(use sum(seq(in calc)

or use RIEMANN prgm

option 2:



$\Delta x = \frac{3}{5}$

Rt End pts: $\frac{3}{5}, \frac{6}{5}, \frac{9}{5}, \frac{12}{5}, 3$

heights: $f(\frac{3}{5}), f(\frac{6}{5}), f(\frac{9}{5}), f(\frac{12}{5}), f(3)$

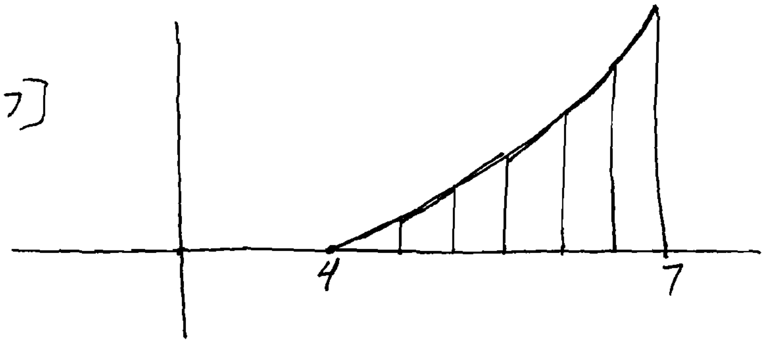
$$A = \frac{3}{5}\left[f(\tfrac{3}{5}) + f(\tfrac{6}{5}) + f(\tfrac{9}{5}) + f(\tfrac{12}{5}) + f(3)\right]$$

$$\frac{3}{5}\left[8.64 + 7.56 + 5.76 + 3.24 + 0\right] = \boxed{15.12\ u^2}$$

(2) $f(x) = x^3 - 4x^2 = 0 \quad [4,7]$

$x^2(x-4) = 0$

$x = 0 \quad x = 4$



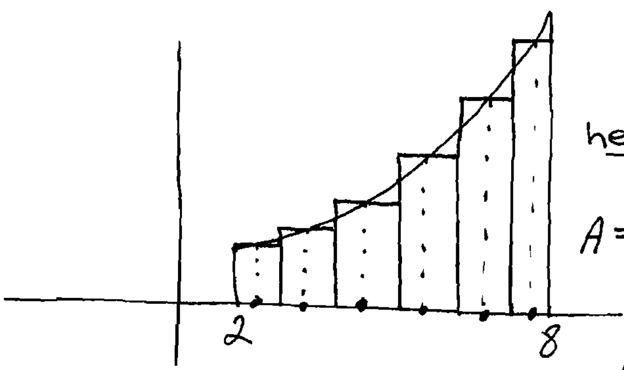
$\Delta x = \frac{7-4}{6} = \frac{1}{2}$

End Pts: $4, 4.5, 5, 5.5, 6, 6.5, 7$

$A = \frac{1}{2}\left[f(4) + 2f(4.5) + 2f(5) + 2f(5.5) + 2f(6) + 2f(6.5) + f(7)\right]$

$\boxed{A = 165.8125 \ u^2}$ ← used RIEMANN to calculate

(3) $f(x) = x^2 \quad [2, 8] \quad \Delta x = \frac{8-2}{6} = 1$



Mid Pts: $2.5, 3.5, 4.5, 5.5, 6.5, 7.5$

heights: $f(2.5), f(3.5), f(4.5), f(5.5), f(6.5), f(7.5)$

$A = 1\left[6.25 + 12.25 + 20.25 + 30.25 + 42.25 + 56.25\right]$

$\boxed{A = 167.5 \ u^2}$

(4) To graph:

$f(1) = 4$

$f(9) = \frac{10}{3}$

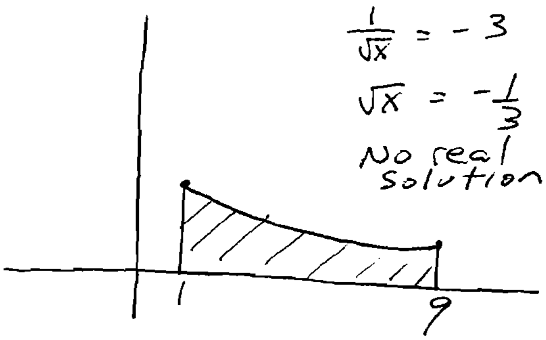
check for zeros:

$\frac{1}{\sqrt{x}} + 3 = 0$

$\frac{1}{\sqrt{x}} = -3$

$\sqrt{x} = -\frac{1}{3}$

No real solution



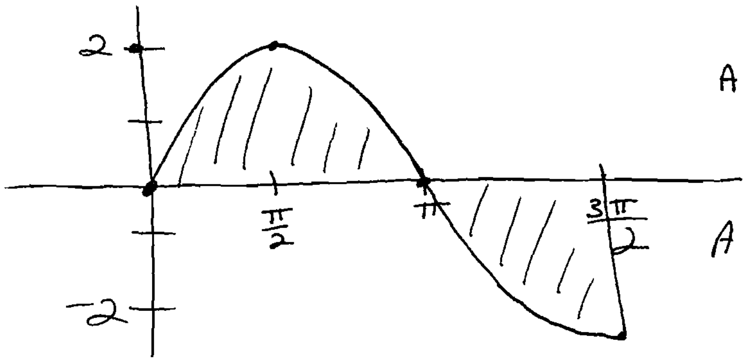
$A = \int_1^9 \left(\frac{1}{\sqrt{x}} + 3\right)dx = \int_1^9 \left(x^{-\frac{1}{2}} + 3\right)dx =$

$2x^{\frac{1}{2}} + 3x \Big|_1^9 = \left(2(9)^{\frac{1}{2}} + 3(9)\right) - \left(2(1)^{\frac{1}{2}} + 3(1)\right)$

$(6 + 27) - (2 + 3)$

$\boxed{28 \ u^2}$

⑤ $f(x) = 2\sin(x) \quad \left[0, \frac{3\pi}{2}\right]$



$$A = \int_0^{\pi} 2\sin(x)\,dx + \left| \int_{\pi}^{\frac{3\pi}{2}} 2\sin(x)\,dx \right|$$

$$A = -2\cos(x)\Big|_0^{\pi} + \left| -2\cos(x)\Big|_{\pi}^{\frac{3\pi}{2}} \right|$$

$$A = \left(-2\cos(\pi) - {}^-2\cos(0)\right) + \left| -2\cos\left(\frac{3\pi}{2}\right) - {}^-2\cos(\pi) \right|$$

$$A = \left[-2(-1) + 2(1)\right] + \left| -2(0) + 2(-1) \right|$$

$$A = 4 + 2 = \boxed{6\ u^2}$$

⑥ $$\int_1^{\frac{e}{2}} \frac{-3}{2x}\,dx = \frac{-3}{2}\int_1^{\frac{e}{2}} \frac{1}{x}\,dx = \frac{-3}{2} \ln|x| \Big|_1^{\frac{e}{2}}$$

$$-\frac{3}{2}\left[\ln\left(\frac{e}{2}\right) - \ln(1)\right] = \frac{-3}{2}\left[\ln\left(\frac{e}{2}\right) - 0\right]$$

$$\boxed{-\frac{3}{2}\ln\left(\frac{e}{2}\right)}$$

(7) $V(t) = 3^{2x-4} - 6$

Total Distance: $\int_0^6 \left|3^{2x-4} - 6\right| dx = \boxed{2978.3696 \text{ ft}}$

Total Distance (other option):

- $V(t) = 0$

$t = 2.8155 \text{ sec}$

$$\left|\int_0^{2.8155} V(t)\,dt\right| + \int_{2.8155}^{6} V(t)\,dt$$

$$14.1677 + 2964.2019 = \boxed{2978.3696 \text{ ft}}$$

Displacement:

$$\int_0^6 V(t)\,dt = \boxed{2950.0342 \text{ ft}}$$

8 $\frac{dy}{dx} = \frac{4x^2 y}{\sqrt{x}}$; $f(3) = -5$

$$y^{-1}\,dy = \frac{4x^2}{x^{1/2}}\,dx$$

$$\int \frac{1}{y}\,dy = \int 4x^{\frac{3}{2}}\,dx$$

$$\ln|y| = \frac{8}{5}x^{\frac{5}{2}} + C$$

$$\ln|\cancel{(}-5| = \frac{8}{5}(3)^{5/2} + C$$

$$1.6094 = 24.9415 + C$$

$$C = -23.3321$$

$$\ln|y| = \frac{8}{5}x^{5/2} - 23.3321$$

$$\boxed{y = e^{\left(\frac{8}{5}x^{5/2} - 23.3321\right)}}$$

⑨ $f(x) = -6x^2 + 12x + 18 \; ; \; [0, 4]$

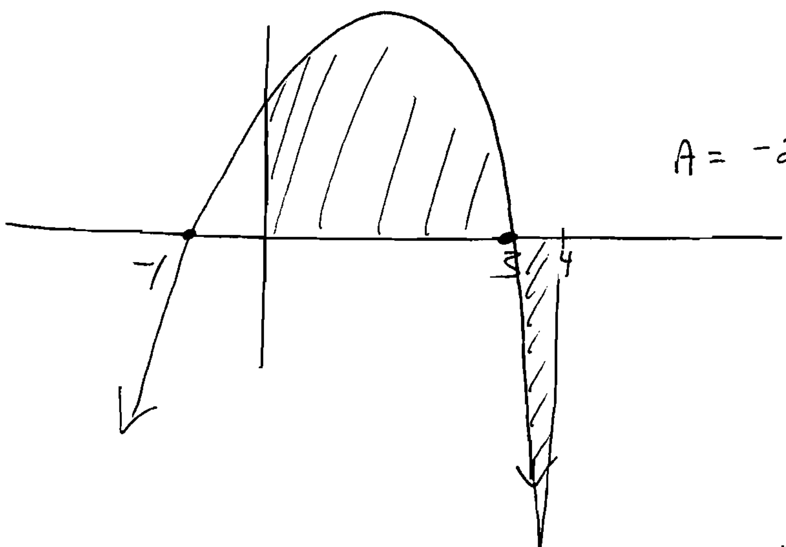
$-6(x^2 - 2x - 3) = 0$

$-6(x+1)(x-3) = 0$

$\cancel{x = -1} \quad x = 3$

$$A = \int_0^3 (-6x^2 + 12x + 18)\,dx + \left| \int_3^4 (-6x^2 + 12x + 18)\,dx \right|$$

$$A = -2x^3 + 6x^2 + 18x \Big|_0^3 + \left| -2x^3 + 6x^2 + 18x \Big|_3^4 \right|$$



$$A = \left[\left(-2(3)^3 + 6(3)^2 + 18(3)\right) - (0)\right] + \left| \left(-2(4)^3 + 6(4)^2 + 18(4)\right) - \left(-2(3)^3 + 6(3)^2 + 18(3)\right) \right|$$

$$A = (-54 + 54 + 54) + \left| (-128 + 96 + 72) - (-54 + 54 + 54) \right|$$

$$54 + |40 - 54|$$

$$54 + |-14| = \boxed{68 \; u^2}$$

(10) $\int_2^8 f(x)\,dx = 6$ & $\int_2^8 g(x)\,dx = 14$

$$\int_2^8 \left(f(x) - 3g(x)\right)dx = \int_2^8 f(x)\,dx - 3\int_2^8 g(x)\,dx$$

$6 - 3(14)$

$6 - 42$

$\boxed{-36}$

(11) $\int_8^{17} \frac{x^2}{\sqrt{3x-5}}\,dx$

$u = 3x - 5 \qquad dx = \frac{du}{3}$

$du = 3\,dx \qquad x = \frac{u+5}{3}$

$$\int_{19}^{46} \left(\frac{u+5}{3}\right)^2 u^{-\frac{1}{2}} \frac{du}{3} = \frac{1}{3}\int_{19}^{46} \left(\frac{u^2+10u+25}{9}\right) u^{-\frac{1}{2}}\,du =$$

$$\frac{1}{27}\int_{19}^{46} \left(u^{\frac{3}{2}} + 10u^{\frac{1}{2}} + 25u^{-\frac{1}{2}}\right)du = \frac{1}{27}\left[\frac{2}{5}u^{5/2} + \frac{20}{3}u^{3/2} + 50u^{\frac{1}{2}}\Big|_{19}^{46}\right]$$

$$\frac{1}{27}\left[\left(\frac{2}{5}(46)^{5/2} + \frac{20}{3}(46)^{3/2} + 50(46)^{\frac{1}{2}}\right) - \left(\frac{2}{5}(19)^{5/2} + \frac{20}{3}(19)^{3/2} + 50(19)^{\frac{1}{2}}\right)\right]$$

use calculator: plug original function into y_1 & use $\boxed{2^{nd}}$ $\boxed{calc}$ $\boxed{\#7}$ from 8 to 17.

$\boxed{\text{Answer} = 250.374}$

(12) $a(t) = -32$

(A) $v(t) = \int a(t)\,dt$

$v(t) = -32t + c$

$19 = -32(3) + c$

$19 = -96 + c$

$+96 \qquad +96$

$115 = c$

$$\boxed{v(t) = -32t + 115}$$

(B) $v(t) = 0$

$-32t + 115 = 0$

$$\boxed{t = 3.5938 \text{ sec}}$$

(C) $\int_0^6 \left| -32t + 115 \right| dt = \boxed{299.2818 \text{ ft}}$

-or-

$v(t) = 0$
$t = 3.5938$ $\rightarrow$ $\int_0^{3.5938} v(t)\,dt + \left| \int_{3.5938}^{6} v(t)\,dt \right|$

$206.6406 + 92.6406$

$$\boxed{299.2812 \text{ ft}}$$